The Quantum Metrological Triangle Experiment

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Abstract: We present our experimental set-up and discuss the results obtained with the quantum metrological triangle (QMT) experiment. This experiment consists in realizing Ohm's law with the three effects used and investigated in quantum electrical metrology: the Josephson effect (JE), the quantum Hall effect (QHE) and the single electron tunneling effect (SET). The aim is to check the consistency of the phenomenological constants K_J, R_K and Q_X associated with these effects and theoretically expressed with the fundamental constants *e* and *h* (elementary charge and Planck constant, respectively). Such an experiment is a contribution for a new definition of the International System of Units (SI). Also, the obtained results are a first step towards a determination of *e*.

Keywords: electrical metrology, low current measurements, cryogenic current comparator, quantum metrological triangle, quantum current standard.

1. EXPERIMENTAL DESCRIPTION

In the framework of the redefinition of the SI base units in term of a reduced set of fundamental constants, the quantum metrological triangle is a key experiment [1-3] since the challenge is to redefine the whole electrical units from only two constants (*h* and *e*). The basic principle is represented in figure 1 and consists in applying a "quantum" Ohm's law [2]:

$$V_J = R_H . I_{SET} \tag{1}$$

V_J and R_H denote the Josephson voltage and the Hall resistance respectively. I_{SET} corresponds to a current generated by a SET device. This is the direct way for closing the QMT.

Another method is the indirect way which consists in developing a quantum capacitance standard from SET devices, called Electron Counting Capacitance Standard (ECCS). This experiment firstly implemented by the NIST allowed to successfully closing the QMT with a relative standard uncertainty of 9.2 parts in 10^7 [4]. The principle is simply based on the natural definition of capacitance. The experimental set-up is completely described in [4].

In our experiment, the SET devices are metallic 3-junctions single electron pumps in aluminium consisted of two gate electrodes and on-chip resistors at the ends in order to reduce the co-tunneling effect [5-6]. The quantized current generated by this pump is theoretically equal to $e.f_{SET}$ (f_{SET} denotes the pumping frequency applied to the gates) and is measured through a cryogenic current comparator (CCC), which allows to amplify the low SET current with a metrological accuracy.

Practically, the closure of the QMT is a measurement of the product $R_K K_J Q_X$, theoretically equal to 2 ($K_J = 2e/h$ is the Josephson constant, $R_K = h/e^2$ the von Klitzing constant and $Q_X = e$ an estimate of the electron charge). Checking the equality $R_K K_J Q_X = 2$ at an uncertainty level of 1 part in 10^8 is the ultimate aim of this experiment and will be a significant contribution to the redefinition of electrical units from (*h*,*e*).

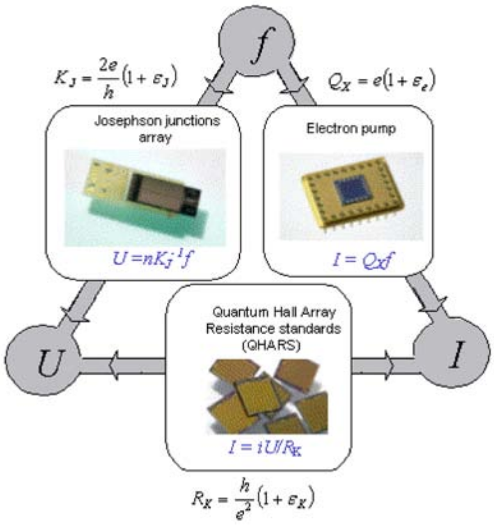

Fig. 1. Principle of the QMT. ε_J , ε_K and ε_e represent the correction factors on K_J, R_K, Q_X.

Our implementation of the direct closure of the QMT is shown in figure 2: the current supplied by the SET source is amplified, then feeds a resistor R_{cal} calibrated against R_K by means of the QHE and the voltage induced at the terminals of the resistor is compared with the voltage $V_J = n_J f_J / K_J$ generated by a Josephson array voltage standard (using n_J junctions irradiated by an RF electromagnetic wave with the frequency f_J). The maximum frequency at which the SET device can be driven is limited to 100 MHz, which corresponds to a quantized current of around 16 pA. This very weak current is amplified and measured through the

CCC. The CCC is a metrological tool with high performance initially developed for the very accurate comparison of resistances. It can be also used as an amplifier with a very accurate known gain [7-8]. The QMT's CCC consists of two windings of N_1 = 20000 turns and N_2 = 1 turn embedded in a superconducting toroïdal shield associated with a DC SQUID, and its gain is exactly equal to the winding ratio ($G = N_1/N_2$). The first winding is directly connected to the pump and is flowed by the quantized current I_{SET} while the second winding is fed by a feedback current $I_{FB} = G.I_{SET}$ supplied by a home-made external stable current source.

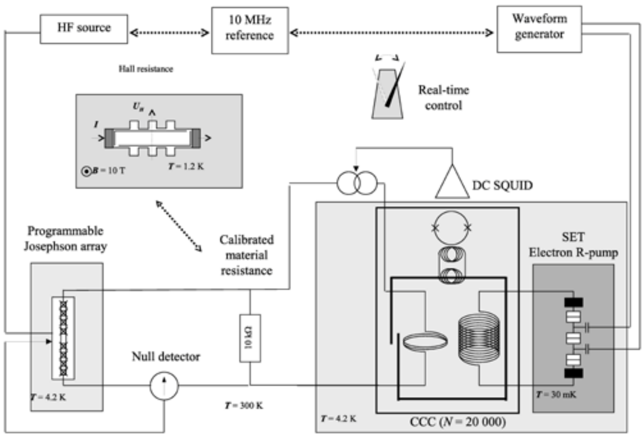

Fig. 2. Our experimental set-up for the direct closure of QMT.

In practice, the relation (1) becomes:

$$V_J = R_{cal}.G.I_{SET} \tag{2}$$

Preliminary results obtained with the complete QMT set-up have demonstrated the feasibility of the experiment. The null detector shown in figure 2 measures the difference between the Josephson voltage and the voltage drop at the terminals of the calibrated resistor, $V_d = V_J - V_R$, with $V_R = R_{cal}.I_{FB}$. From this datum and for testing the measurement bench, the quantity $\Delta e/e$ has been determined in the following way:

$$\frac{\Delta e}{e} = \frac{Q_x - e_{CODATA}}{e_{CODATA}} \tag{3}$$

where

$$Q_x = \left(\frac{1}{G.f_{SET}.R}\right).\left(\frac{n_J f_J}{K_J} - V_d\right) \tag{4}$$

$\Delta e/e$ corresponds to the discrepancy between the CODATA value of the elementary charge [9] and the single charge Q_X of our SET pump. In other words, this measurement allows to estimate the exactness of the SET device or the deviation from the current quantization ($e.f_{SET}$).

2. RESULTS

The various parameters of the SET pump have been estimated from the experimental stability diagram as described in a previous paper [6]. The three junction capacitances C_L, C_m, C_R (left, middle and right resp.) have been found to be C_L = 115 aF, C_m = 80 aF, C_R = 115 aF. The two gate electrodes controlling the charging states of the islands have capacitances of C_{g1} = 35 aF, C_{g2} = 30 aF. The parameters characterizing the cross-talking effect due to the proximity of islands, defined in [6], have been found to be f_1 = 0.3 and f_2 = 0.25. A white noise level close to 3 fA/Hz$^{1/2}$ has been measured with the complete measuring system at 1 Hz. A large series of current plateau measurements (I_{pump}/V_{bias}) has been performed with the complete experimental set-up (figure 2) in the frequency range 10–100 MHz by measuring the voltage at the R_{cal} terminals by means of a multimeter (figure 3). Negative current steps have been carried out by phase shifting the second harmonic signal [6].

As expected, when the frequency is increased the plateau width decreases and the centre of the plateau where the co-tunnelling is minimized shifts towards larger bias voltage.

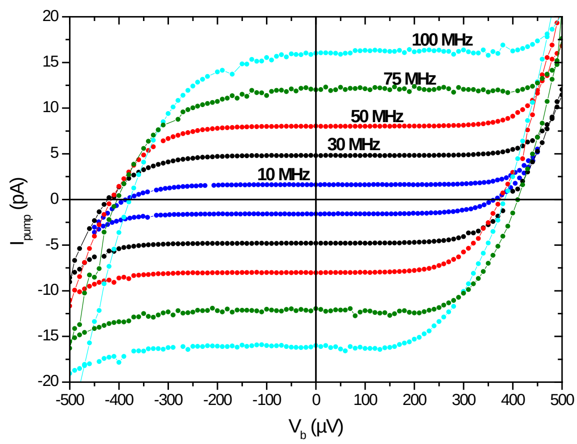

Fig. 3. A series of current plateaus in the frequency range 10–100 MHz.

As shown in figure 4 (blow-up of two current steps for 10 MHz and 30 MHz pumping frequencies) the current plateau is flat over a relatively large bias voltage range, 350 µV for 30 MHz and 390 µV for 10 MHz, within the noise floor ΔI_{nf} = 40 fA. These results are similar to those obtained in internal feedback mode where the output current of the SQUID electronics is fedback directly to the SQUID and not in the secondary winding of the CCC [6].

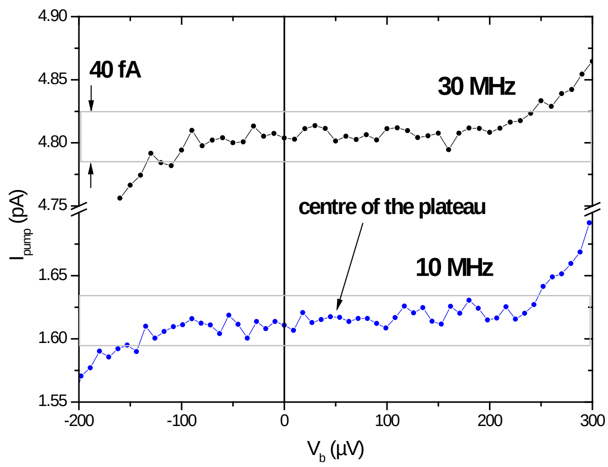

Fig. 4. Blow-up of two current plateaus obtained at 10 MHz and 30 MHz pumping frequencies. The hatched parts represent the 40 fA noise for a 5 minutes measuring time.

Then, long-term measurements have been performed in order to reduce random uncertainty. The bias voltage is adjusted in such a way that the generated current corresponds to the center of the plateau. Investigations about the time domain stability of a R-pump have demonstrated that $1/f$ noise occurs very early after roughly 100 s [6]. The appearance of a $1/f$ regime displays high correlations among the measurable quantities and limits the reduction of the random uncertainty. The 1/f noise can be avoided by periodically reversing the SET current with modifying, for instance, the phase shift between the two gate harmonic signals.

As a consequence, the complete system including the stable current source and the DC SQUID has to be especially designed for quick (roughly at the rate of 1 Hz) periodic inversions of the current. The synchronization between the Josephson array and the current supplied by the SET device is ensured by Digital Analogical Converters (DAC) driven by a computer (not represented in the figure 2).

From measurements carried out over measuring time as long as 10 hours, Allan standard deviations have been calculated for two different frequencies and reported in figure 5. Five (eight, resp.) junctions of the Josephson array have been biased for balancing the potential drop induced by the current of the pump driven at 23.55 MHz (37.69 MHz, resp.).

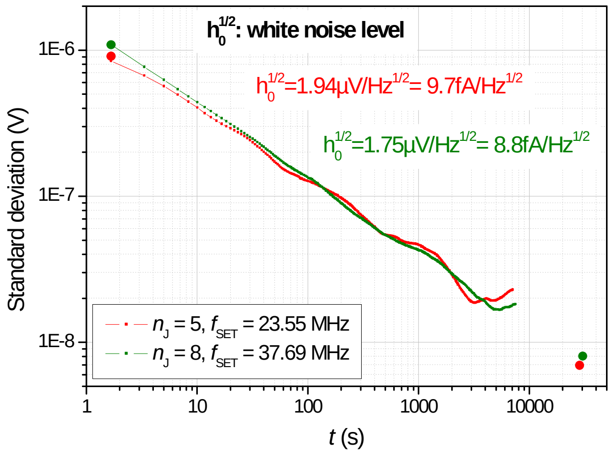

Fig. 5. Allan standard deviations (dots and lines) and experimental standard deviation of the mean (large points) calculated for two different frequencies f_{SET} = 23.55 MHz and 37.69 MHz corresponding to 5 and 8 biased junctions of the Josephson array respectively.

The Allan standard deviation, firstly developed by the time and frequency community, is used for describing the noise nature over long time ranges and for revealing the possible correlations among data. Moreover, it is a convenient statistical tool because in white noise regime, *i.e.* when the data are uncorrelated, the Allan variance and the experimental variance of the mean are unbiased estimates of the theoretical variance. As a consequence, the experimental standard deviation of the mean (ESDM), recommended in the GUM for characterizing the type A uncertainty, can directly be read on the Allan deviation curve shown in figure 5 since this curve varies according to a $\tau^{-1/2}$-law distinguishing a white noise regime.

From measurements of V_d in readout of the null detector, the quantity $\Delta e/e$, discrepancy between Q_X and the CODATA recommended value of e [9] has been calculated and reported in figure 6. For each value, the ESDM calculated through Allan deviation is given. Within one run corresponding to 6 measurements (figure 5) and with a given adjustment of the parameters, a consistent set of data has allowed to achieve a weighted random uncertainty of 4 parts in 10^6. A similar result (7 parts in 10^6) has been obtained with a second pumping frequency (f_{SET} = 23.55 MHz) and 5 biased Josephson junctions. However, discrepancies as large as 1.5 parts in 10^4 have been measured on the absolute value of SET current from a run to the other (figure 7). Some changes on the experimental set-up (shielding, ground…) are in progress in order to suppress these irreproductibilities.

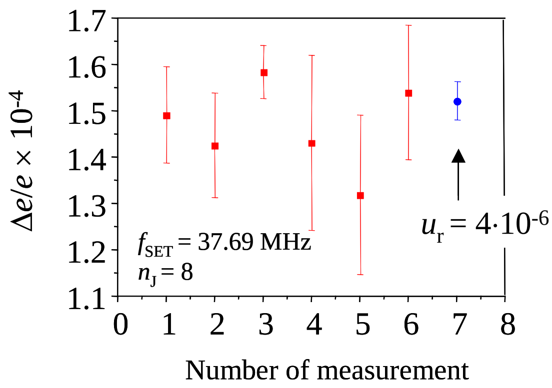

Fig. 6. A series of 6 measurements performed at a 37.69 MHz pumping frequency over a run. The blue point is the weighted mean associated with its relative uncertainty u_r.

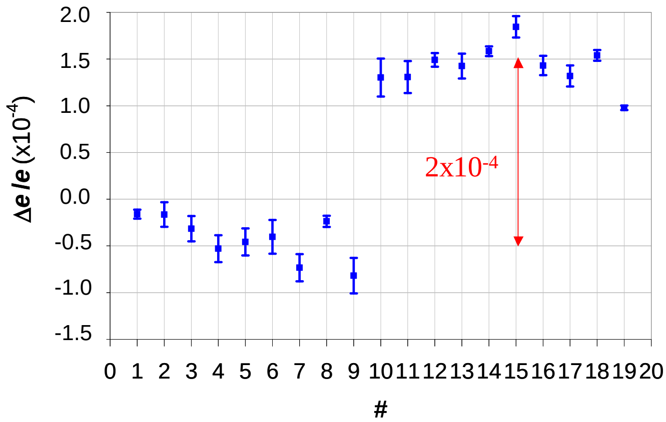

Fig. 7. Discrepancies observed from one run to another for f_{SET} = 37.69 MHz and 8 biased junctions.

3. CONCLUSION

These preliminary results prove that the direct QMT set-up never implemented up to now is operating. The final random uncertainty of 4 ppm is encouraging to hope to close the QMT at a 1 ppm level considered as a first important step in the near future. Above all, the troubles related to the non reproducible measurements must be solved. But we can already claim that the QMT set-up is the best measuring bench for testing the exactness of the current generated by the single electron devices. A new generation of SET devices able to generate currents as high as 1 nA is expected for replacing the aluminium devices [10-11]. Consequently the set-up described in this work will be able to be used to verify the quantization of the current they supply.

Afterwards, checking the equality $R_K K_J Q_X = 2$ with lower uncertainties for preparing the new S.I. induces the use of a new type of CCC and/or implementation of a new SET candidate. For instance, CCCs consisted of more sensitive SQUID are presently developed in our laboratory.

Finally, in case the 10^{-8} targeted uncertainty would be achieved, the set of experiments {QMT, watt balance, calculable capacitor} could be an alternative for replacing the calculation given by the relation $e = [2\alpha h/(\mu_0 c)]^{1/2}$ in the CODATA adjustment by a direct determination of e [6,12].

In summary, in the framework of a modification of the SI, the closure of the QMT will have major implications for electrical units. Indeed, a successful outcome both for the Watt balance and for the QMT would make it possible to link the kilogram to h and the Ampere to e. In this case, a determination of e and a sufficient confidence in the relations $K_J = 2e/h$ and $R_K = h/e^2$ are needed. As shown above, the QMT can strengthen this confidence and allows a determination of the charge quantum Q_X in the SI of units.

AKNOWLEDGMENTS

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